

How the Charge Can Affect the Formation of Gravastars

R. Chan ^{1*} and M.F.A. da Silva ^{2†}

¹ *Coordenação de Astronomia e Astrofísica,*

Observatório Nacional, Rua General José Cristino, 77,

São Cristóvão, CEP 20921-400, Rio de Janeiro, RJ, Brazil

² *Departamento de Física Teórica, Instituto de Física,*

Universidade do Estado do Rio de Janeiro,

Rua São Francisco Xavier 524, Maracanã,

CEP 20550-900, Rio de Janeiro - RJ, Brasil

(Dated: July 8, 2010)

Abstract

In recent work we physically interpreted a special gravastar solution characterized by a zero Schwarzschild mass. In fact, in that case, none gravastar was formed and the shell expanded, leaving behind a de Sitter or a Minkowski spacetime, or collapsed without forming an event horizon, originating what we called a massive non-gravitational object. This object has two components of non zero mass but the exterior spacetime is Minkowski or de Sitter. One of the component is a massive thin shell and the other one is de Sitter spacetime inside. The total mass of this object is zero Schwarzschild mass, which characterizes an exterior vacuum spacetime. Here, we extend this study to the case where we have a charged shell. Now, the exterior is a Reissner-Nordström spacetime and, depending on the parameter $\omega = 1 - \gamma$ of the equation of state of the shell, and the charge, a gravastar structure can be formed. We have found that the presence of the charge contributes to the stability of the gravastar, if the charge is greater than a critical value. Otherwise, a massive non-gravitational object is formed for small charges.

PACS numbers: 98.80.-k, 04.20.Cv, 04.70.Dy

*Electronic address: chan@on.br

†Electronic address: mfasnic@gmail.com

I. INTRODUCTION

As alternatives to black holes, gravastars have received some attention recently [1][2], partially due to the tight connection between the cosmological constant and a currently accelerating universe [3], although very strict observational constraints on the existence of such stars may exist [4].

The pioneer model of gravastar was proposed by Mazur and Mottola (MM) [5]. After this work, Visser and Wiltshire (VW) [6] reduced the number of shells of the original model from five to three. The interior was described by the de Sitter metric, while the exterior by the Schwarzschild metric. Between both there is a thin shell constituted by stiff fluid, which is located in a such way that it is outside of the Schwarzschild event horizon and inside of the de Sitter one, in order to eliminate both horizons from the whole spacetime. They also pointed out that there are two different types of stable gravastars which are stable gravastars and "bounded excursion" gravastars. The first one represents a stable structure already formed, while the second one is a system with a shell which oscillates around a equilibrium position which can loose energy and to stabilize at the end.

Recently we have done an extensive study on the problem of the stability of gravastars. The first model [7] consisted of an internal de Sitter spacetime, a dynamical infinitely thin shell of stiff fluid, and an external Schwarzschild spacetime, as proposed by VW [6]. We have shown explicitly that the final output can be a black hole, a "bounded excursion" stable gravastar, a Minkowski, or a de Sitter spacetime, depending on the total mass m of the system, the cosmological constant Λ , and the initial position R_0 of the dynamical shell. Therefore, we have shown, for the first time in the literature, that although it does exist a region of the space of the initial parameters where it is always formed stable gravastars, it still exists a large region of this space where we can find black hole formation. Then, we conclude that gravastar is not an alternative model to black hole as it was originally proposed by VW models [6].

In the second paper [8], we have generalized the previous work on the problem of stable gravastars considering an equation of state $p = (1 - \gamma)\sigma$ for the shell, instead of only using a stiff fluid ($\gamma = 0$). We have found that stable gravastars can be formed even for $\gamma \neq 0$, since $\gamma < 1$, generalizing the gravastar models proposed until now. We also have confirmed the previous results, i.e., that both gravastars and black holes can be formed, depending on

the initial parameters.

In the third work [9], we have generalized the former one considering now an interior constituted by an anisotropic dark energy fluid. We have again confirmed the previous results, i.e., that both gravastars and black holes can be formed, depending on the initial parameters. It is remarkable that for this case we have an interior filled by a physical matter, instead of a de Sitter vacuum. Thus, it is similar to phantom energy star models.

In the fourth paper [10], we generalized a previous model of gravastar consisted of an internal de Sitter spacetime, a dynamical infinitely thin shell with an equation of state, but we have considered an external de Sitter-Schwarzschild spacetime. We have found that the exterior cosmological constant imposes a limit to the gravastar formation, i.e., the exterior cosmological constant must be smaller than the interior cosmological constant.

In another work [11], we investigated a particular solution, which emerges from the gravastar studies, with zero Schwarzschild mass, which implies in a non-gravitational object. This kind of structure can be possible, since the gravitational mass depends on the trace of the energy momentum tensor, instead of the energy density only. As the inner region is filled by dark energy, there is an equilibrium between the repulsive gravitational effect (due to the inner region) and the attractive one (due to the thin shell) on all the test particles in the exterior, vanishing the gravitational interactions. So, the object that we have studied is similar to a point-like topological defect, since the vacuum solutions do not describe all the spacetime.

Before the present work, Carter [12] studied spherically symmetric gravastar solutions which possess an (anti) de Sitter interior and a Reissner-Nordström exterior. He followed the same approach that Visser and Wiltshire took in their work [6] assuming a potential $V(a)$ and then finding the equation of state of the shell. He have found a wide range of parameters which allows stable gravastar solutions, and presented the different qualitative behaviors of the equation of state for these parameters. Gravastar with charge has been also studied by others authors [13].

It is the aim of this work to generalize that solution for zero Schwarzschild mass, but now, considering a charged shell, in order to investigate the effect of the charge on the stability of the gravastar formation.

The paper is organized as follows. In Section II we present the charged gravastar model. In Section III a discussion on the definition of the gravitational mass of the charged gravastar

is done. In Section IV we find the potential of the gravastar's shell and we study two special cases, that are: i) dust fluid ($\gamma = 1$) and ii) stiff fluid ($\gamma = 0$). Finally, in Section V we present the final remarks.

II. CHARGED GRAVASTAR MODEL

The interior spacetime is described by the de Sitter metric given by

$$ds_i^2 = -f dt^2 + \frac{1}{f} dr^2 + r^2 d\Omega^2, \quad (1)$$

where $f = 1 - (r/L_i)^2$, $L_i = \sqrt{3/\Lambda_i}$ and $d\Omega^2 = d\theta^2 + \sin^2(\theta)d\phi^2$.

The exterior spacetime is given by a Reissner-Nordström metric, which characterizes a vacuum charged exterior spacetime

$$ds_e^2 = -h dv^2 + \frac{1}{h} d\mathbf{r}^2 + \mathbf{r}^2 d\Omega^2, \quad (2)$$

where $h = 1 - \frac{2m}{\mathbf{r}} + \left(\frac{q}{\mathbf{r}}\right)^2$.

In order to follow the pioneer model of gravastar proposed by Mazur and Mottola (MM) [5] and Visser and Wiltshire (VW) [6], we have assumed an inner spacetime with cosmological constant. The presence of a cosmological constant only in the inner spacetime can be understood in the context where it emerges from a phase transition of the quantum vacuum near or at the place where the event horizon is expected to form. Thus, it does not need to be present in the whole spacetime.

The metric of the hypersurface do the shell is given by

$$ds_\Sigma^2 = -d\tau^2 + R^2(\tau) d\Omega^2. \quad (3)$$

Since $ds_i^2 = ds_e^2 = ds_\Sigma^2$ then $r_\Sigma = \mathbf{r}_\Sigma = R$, and besides

$$\dot{t}^2 = \left[f - \frac{1}{f} \left(\frac{\dot{R}}{\dot{t}} \right)^2 \right]^{-1} = \left[1 - \left(\frac{R}{L_i} \right)^2 \right]^{-2} \left[1 - \left(\frac{R}{L_i} \right)^2 + \dot{R}^2 \right], \quad (4)$$

and

$$\dot{v}^2 = \left[h - h^{-1} \left(\frac{\dot{R}}{\dot{v}} \right)^2 \right]^{-1} = \left[1 - \frac{2m}{\mathbf{r}} + \left(\frac{q}{\mathbf{r}} \right)^2 \right]^{-2} \left[\dot{R}^2 + 1 - 2\frac{m}{R} + \left(\frac{q}{R} \right)^2 \right], \quad (5)$$

where the dot represents the differentiation with respect to τ .

Thus, the interior and exterior normal vector are given by

$$n_\alpha^i = (-\dot{R}, \dot{t}, 0, 0), \quad n_\alpha^e = (-\dot{R}, \dot{v}, 0, 0). \quad (6)$$

The interior and exterior extrinsic curvature are given by

$$K_{\tau\tau}^i = -[(3L_i^4 \dot{R}^2 - L_i^4 \dot{t}^2 + 2L_i^2 R^2 \dot{t}^2 - R^4 \dot{t}^2) R \dot{t} - (L_i + R)(L_i - R)(\dot{R}\ddot{t} - \ddot{R}\dot{t}) L_i^4] \times \\ (L_i + R)^{-1} (L_i - R)^{-1} L_i^{-4} \quad (7)$$

$$K_{\theta\theta}^i = \dot{t} (L_i + R)(L_i - R) L_i^{-2} R \quad (8)$$

$$K_{\phi\phi}^i = K_{\theta\theta}^i \sin^2(\theta), \quad (9)$$

$$K_{\tau\tau}^e = - \left\{ \left[(2mR\dot{v} - q^2\dot{v} + R^2\dot{R} - R^2\dot{v})(2mR\dot{v} - q^2\dot{v} - R^2\dot{R} - R^2\dot{v}) - R^4\dot{R}^2 - R^4\dot{v}^2 \right] \times \right. \\ \left. (mR - q^2)\dot{v} - (q^2 + R^2 - 2mR)(\dot{R}\ddot{v} - \ddot{R}\dot{v}) R^5 \right\} (q^2 + R^2 - 2mR)^{-1} R^{-5}, \quad (10)$$

$$K_{\theta\theta}^e = \dot{v} (q^2 + R^2 - 2mR) R^{-1}, \quad (11)$$

$$K_{\phi\phi}^e = K_{\theta\theta}^e \sin^2(\theta). \quad (12)$$

Since we have [14]

$$[K_{\theta\theta}] = K_{\theta\theta}^e - K_{\theta\theta}^i = -M, \quad (13)$$

where M is the mass of the shell, thus

$$M = \dot{t} (L_i + R)(L_i - R) \frac{R}{L_i^2} - \dot{v} (q^2 + R^2 - 2mR) \frac{1}{R}. \quad (14)$$

Then, substituting equations (4) and (5) into (14) we get

$$M - R \left[1 - \left(\frac{R}{L_i} \right)^2 + \dot{R}^2 \right]^{1/2} + R \left[1 - \frac{2m}{R} + \dot{R}^2 + \left(\frac{q}{R} \right)^2 \right]^{1/2} = 0. \quad (15)$$

Solving the equation (15) for $\dot{R}^2/2$ we obtain the potential $V(R, m, L_i, q)$. In order to keep the ideas of our work [8] as much as possible, we consider the thin shell as consisting of a fluid with a equation of state, $\vartheta = (1 - \gamma)\sigma$, where σ and ϑ denote, respectively, the surface energy density and pressure of the shell and γ is a constant. The equation of motion of the shell is given by [14]

$$\dot{M} + 8\pi R \dot{R} \vartheta = 4\pi R^2 [T_{\alpha\beta} u^\alpha n^\beta] = \pi R^2 \left(T_{\alpha\beta}^e u_e^\alpha n_e^\beta - T_{\alpha\beta}^i u_i^\alpha n_i^\beta \right), \quad (16)$$

where u^α is the four-velocity and n^β is its orthogonal vector. Since the interior region is constituted by a vacuum with cosmological constant and the exterior correspond to a vacuum with an electromagnetic field, characterized by the Reissner-Nordström spacetime, it is easy to show that

$$\dot{M} + 8\pi R\dot{R}(1 - \gamma)\sigma = 0, \quad (17)$$

since [15]

$$T_{\alpha\beta}^e = F_{\alpha\lambda}F_{\beta}^{\lambda} + \frac{1}{4}F_{\lambda\nu}F^{\lambda\nu} = \frac{q^2}{2\mathbf{r}^4} \left(h\delta_{\alpha}^v\delta_{\beta}^v - h^{-1}\delta_{\alpha}^{\mathbf{r}}\delta_{\beta}^{\mathbf{r}} + \mathbf{r}^2\delta_{\alpha}^{\theta}\delta_{\beta}^{\theta} + \mathbf{r}^2\sin^2\theta\delta_{\alpha}^{\phi}\delta_{\beta}^{\phi} \right), \quad (18)$$

where $F_{\alpha\lambda}$ is the Maxwell tensor.

Since $\sigma = M/(4\pi R^2)$ we can solve equation (17) giving

$$M = kR^{2(\gamma-1)}, \quad (19)$$

where k is an integration constant.

Substituting equation (19) into $V(R, m, L_i, q)$, we obtain the general expression for the potential,

$$\begin{aligned} V(R, m, L_i, q, \gamma) = & \frac{1}{2} - \frac{1}{4} \frac{R^2}{L_i^2} - \frac{1}{8} \frac{R^{10}}{R^{4\gamma} L_i^4} + \frac{1}{4} \frac{q^2}{R^2} - \frac{1}{2} \frac{m}{R} + \frac{1}{2} \frac{R^3 m q^2}{R^{4\gamma}} \\ & - \frac{1}{8} \frac{R^{4\gamma}}{R^6} - \frac{1}{4} \frac{R^6 q^2}{R^{4\gamma} L_i^2} - \frac{1}{8} \frac{R^2 q^4}{R^{4\gamma}} \\ & + \frac{1}{2} \frac{R^7 m}{R^{4\gamma} L_i^2} - \frac{1}{2} \frac{R^4 m^2}{R^{4\gamma}} \end{aligned} \quad (20)$$

where we have redefined the Schwarzschild mass m , the cosmological constants L_i , the charge q and the radius R as $m \equiv mk^{-\frac{1}{2\gamma-3}}$, $L_i \equiv L_i k^{\frac{2}{2\gamma-3}}$, $q \equiv qk^{\frac{2}{2\gamma-3}}$, $R \equiv Rk^{-\frac{1}{2\gamma-3}}$.

III. TOTAL GRAVITATIONAL MASS

In order to study the gravitational effect generated by the two components of the gravastar, i.e., the interior de Sitter and the thin shell in the exterior region, we need to calculate the total gravitational mass of a spherical symmetric system. Some alternative definitions are given by [16],[17] and [18]. Here we consider the Tolman formula for the mass, given by

$$M_G = \int_0^{R_0} \int_{-\pi}^{\pi} \int_0^{2\pi} \sqrt{-g} T_{\alpha}^{\alpha} dr d\theta d\phi, \quad (21)$$

where $\sqrt{-g}$ is the determinant of the metric. For the special case of a thin shell we have

$$M_G = \int_0^{R_0} \int_{-\pi}^{\pi} \int_0^{2\pi} \sqrt{-g} T_{\alpha}^{\alpha} \delta(\mathbf{r} - R_0) d\mathbf{r} d\theta d\phi. \quad (22)$$

Since the energy-momentum tensor of the shell is given by

$$T_{\alpha\beta} = T_{\alpha\beta}^{(F)} + T_{\alpha\beta}^{(EM)}, \quad (23)$$

where the (F) denotes the fluid and (EM) the electromagnetic tensor and

$$T_{\alpha\lambda}^{(EM)} g^{\lambda\beta} = \frac{q^2}{2\mathbf{r}^4} \left(-\delta_{\alpha}^v \delta_v^{\beta} - \delta_{\alpha}^{\mathbf{r}} \delta_{\mathbf{r}}^{\beta} + \delta_{\alpha}^{\theta} \delta_{\theta}^{\beta} + \delta_{\alpha}^{\phi} \delta_{\phi}^{\beta} \right). \quad (24)$$

Thus, the gravitational mass of the thin shell is given by

$$M_G^{shell} = (3 - 2\gamma)M, \quad (25)$$

since $T_{\alpha\beta}^{(EM)} g^{\alpha\beta} = 0$.

For the interior de Sitter (dS) spacetime we have

$$M_G^{(dS)} = -\frac{2}{3}\Lambda_i R_0^3. \quad (26)$$

Then, the de Sitter interior presents a negative gravitational mass, since $\Lambda_i > 0$, in accordance to the its repulsive effect.

Now we can write the total gravitational mass of the gravastar as

$$M_G^{(total)} = M_G^{(shell)} + M_G^{(dS)} = (3 - 2\gamma)M - \frac{2}{3}\Lambda_i R_0^3. \quad (27)$$

This mass also represents the Schwarzschild exterior mass ($m = M_G^{(total)}$) for this gravastar. Thus, it is possible to obtain a physical structure through a combination of these two solutions, which results in a system with $m = 0$, a Reissner-Nordström exterior spacetime, without mass but with charge.

IV. THE POTENTIAL FOR $m = 0$

Firstly, we can see that in this case, the exterior metric has no event horizon. So, the initial radius of the shell must be in the interval $0 \leq R_0 \leq L_i$. For the particular case where the Schwarzschild mass is vanished, $m = 0$, the potential (20) is written as

$$\begin{aligned} V(R, L_i, q, \gamma) = & \frac{1}{2} - \frac{1}{4} \frac{R^2}{L_i^2} - \frac{1}{4} \frac{q^2 R^6}{L_i^2 R^{4\gamma}} + \frac{1}{4} \frac{q^2}{R^2} - \\ & \frac{1}{8} \frac{R^{4\gamma}}{R^6} - \frac{1}{8} \frac{R^{10}}{R^{4\gamma} L_i^4} - \frac{1}{8} \frac{R^2 q^4}{R^{4\gamma}}, \end{aligned} \quad (28)$$

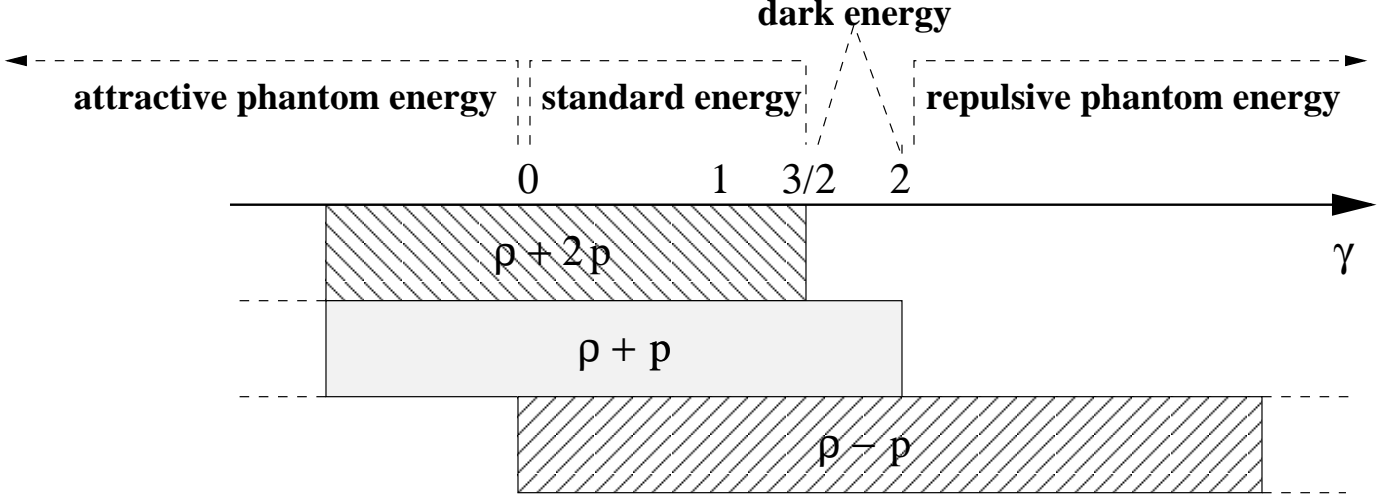


FIG. 1: This is a plot with the different kinds of the shell's matter depending on the parameter γ .

and differentiating the potential we get

$$\begin{aligned} \frac{dV(R)}{dR} = & -\frac{1}{2} \frac{R}{L_i^2} - \frac{3}{2} \frac{q^2 R^5}{L_i^2 R^{4\gamma}} - \frac{1}{2} \frac{q^2}{R^3} + \frac{3}{4} \frac{R^{4\gamma}}{R^7} - \frac{5}{4} \frac{R^9}{R^{4\gamma} L_i^4} \\ & - \frac{1}{4} \frac{R q^4}{R^{4\gamma}} + \frac{R^5 \gamma q^2}{L_i^2 R^{4\gamma}} - \frac{1}{2} \frac{\gamma R^{4\gamma}}{R^7} + \frac{1}{2} \frac{R^9 \gamma}{R^{4\gamma} L_i^4} + \frac{1}{2} \frac{R \gamma q^4}{R^{4\gamma}}. \end{aligned} \quad (29)$$

A second differentiation of the potential furnishes

$$\begin{aligned} \frac{d^2 V(R)}{dR^2} = & -\frac{1}{2L_i^2} + 11 \frac{R^4 \gamma q^2}{L_i^2 R^{4\gamma}} - \frac{15}{2} \frac{q^2 R^4}{L_i^2 R^{4\gamma}} + \frac{3}{2} \frac{q^2}{R^4} + \\ & \frac{13}{2} \frac{\gamma R^{4\gamma}}{R^8} + \frac{19}{2} \frac{R^8 \gamma}{R^{4\gamma} L_i^4} - \frac{45}{4} \frac{R^8}{R^{4\gamma} L_i^4} + \frac{3}{2} \frac{\gamma q^4}{R^{4\gamma}} - \\ & - \frac{1}{4} \frac{q^4}{R^{4\gamma}} - 4 \frac{q^2 R^4 \gamma^2}{L_i^2 R^{4\gamma}} - 2 \frac{R^{4\gamma} \gamma^2}{R^8} - \\ & 2 \frac{R^8 \gamma^2}{R^{4\gamma} L_i^4} - 2 \frac{q^4 \gamma^2}{R^{4\gamma}} - \frac{21}{4} \frac{R^{4\gamma}}{R^8}. \end{aligned} \quad (30)$$

In the following we will consider some particular equations of state, representing some different kinds of energy, shown in the figure 1.

A. Stiff Fluid Shell ($\gamma = 0$):

In order to simplify, firstly we have considered a shell constituted by stiff fluid, as it was done in the original gravastars model, meaning that we choose $\gamma = 0$. Vanishing the first derivative of the potential gives us the critical values of the parameter L_i , or the positive values of L_i^2 which assure that the potential has a profile as shown in the figure 2.

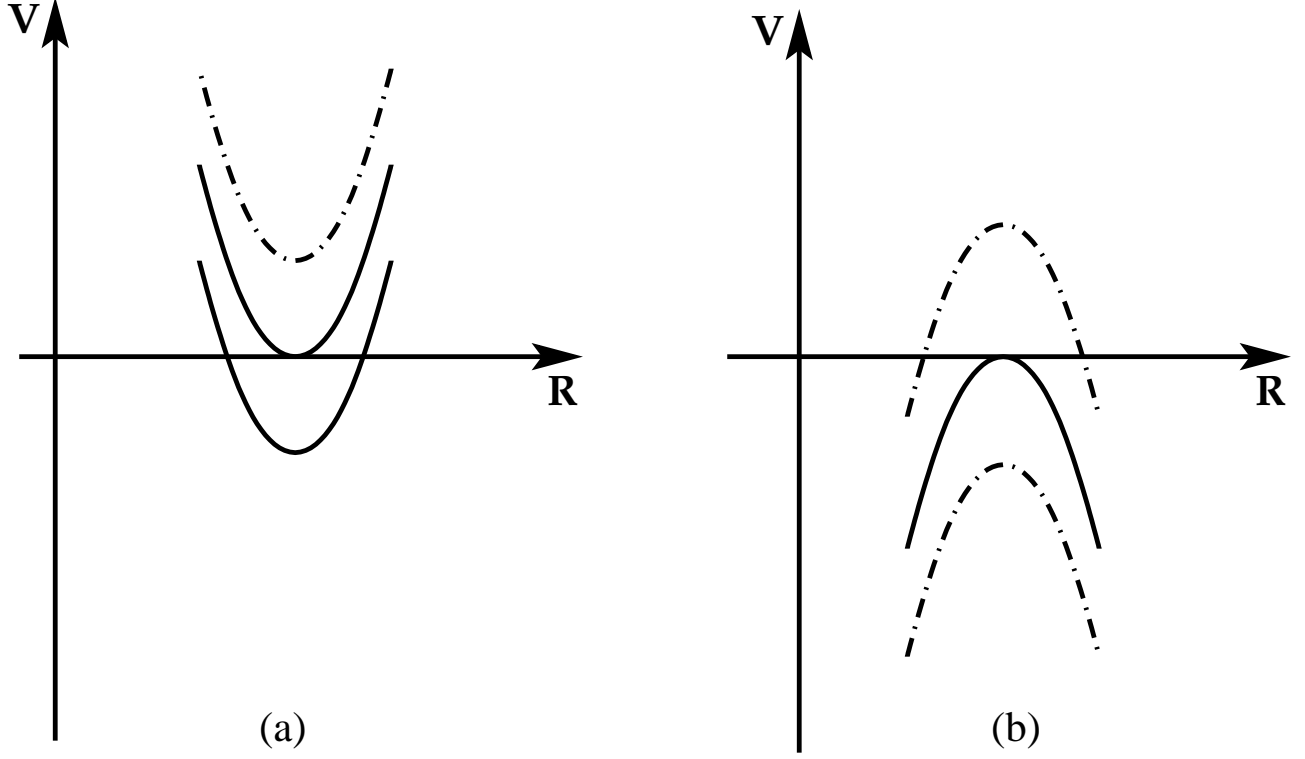


FIG. 2: This plot shows the possible potentials.

Doing this we obtain

$$L_{ic}^2 = -\frac{1}{\Delta_1} (3R^{12}q^2 + R^8 + 2R^8\Delta_2), \quad (31)$$

where $\Delta_1 = 2q^2R^4 - 3 + R^8q^4$ and $\Delta_2 = \sqrt{R^8q^4 - q^2R^4 + 4}$. Substituting L_i^2 by L_{ic}^2 into the second derivative of the potential, we obtain the expression

$$\begin{aligned} \frac{d^2V(R)}{dR^2} = \\ \frac{1}{\Delta_3} \left(-\frac{192}{R^8} + 8R^8q^8 - 96q^4 - \frac{24\Delta_2}{R^8} + 48q^4\Delta_2 - \frac{100q^2\Delta_2}{R^4} + 12R^4q^6\Delta_2 + 28R^4q^6 + \frac{124q^2}{R^4} \right), \end{aligned} \quad (32)$$

where $\Delta_3 = (3q^2R^4 + 1 + 2\Delta_2)^2$.

Then we can see, from figures 3 and 4, that the potential can present a minimum value. From figure 5 we can see that it represents a stable configuration (or oscillating) since its minimum value is not positive.

Substituting L_i by L_{ic} into the potential, and equating it to zero, we obtain the critical values of q , in order to guarantee that the maximum of the potential is zero, which is given

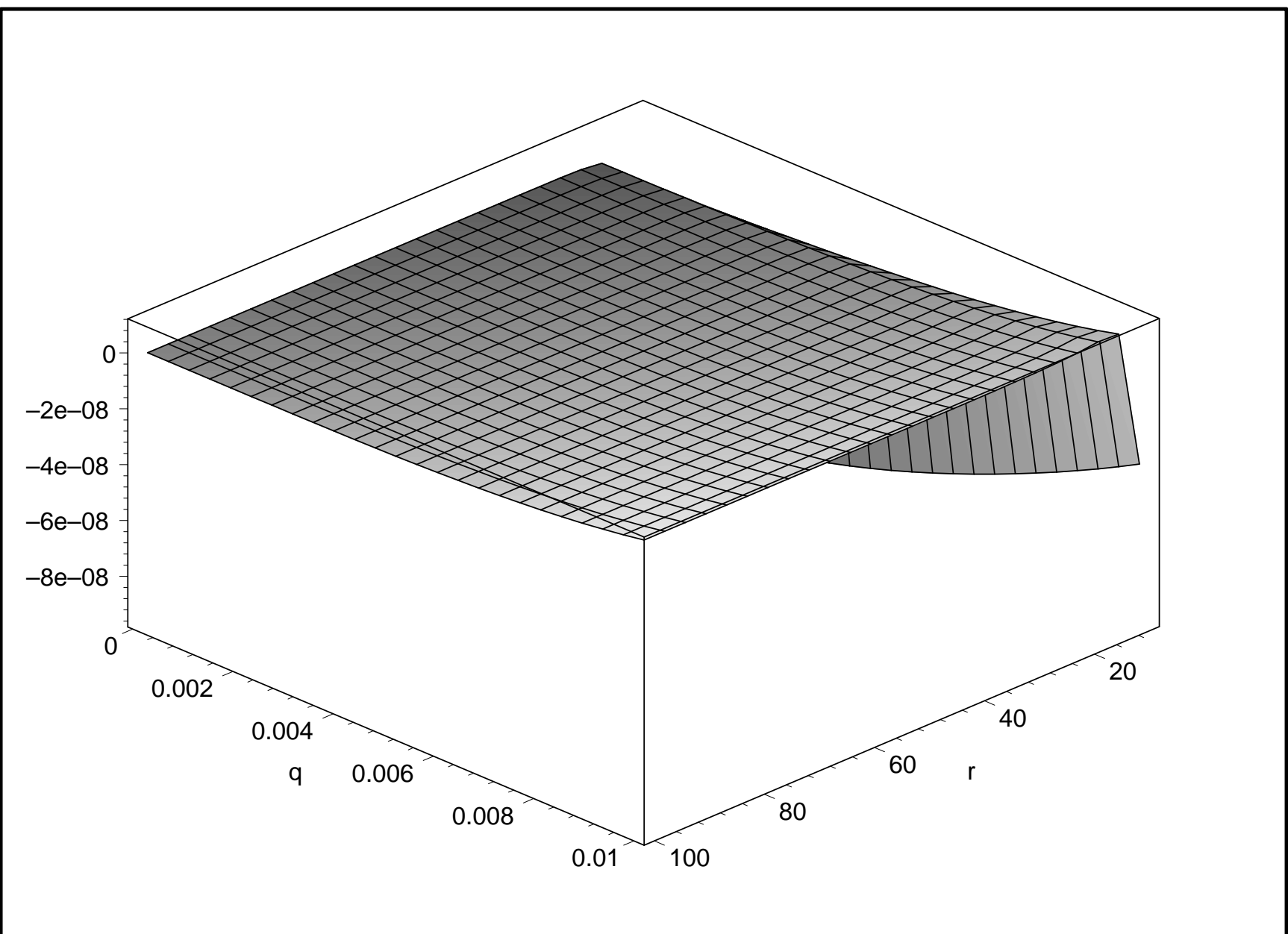


FIG. 3: This plot shows, in terms of R and q , the second derivative of the potential $V(R, q)$, for $\gamma = 0$ and for the intervals $0 \leq q \leq 0.01$ and $15 \leq R \leq 100$.

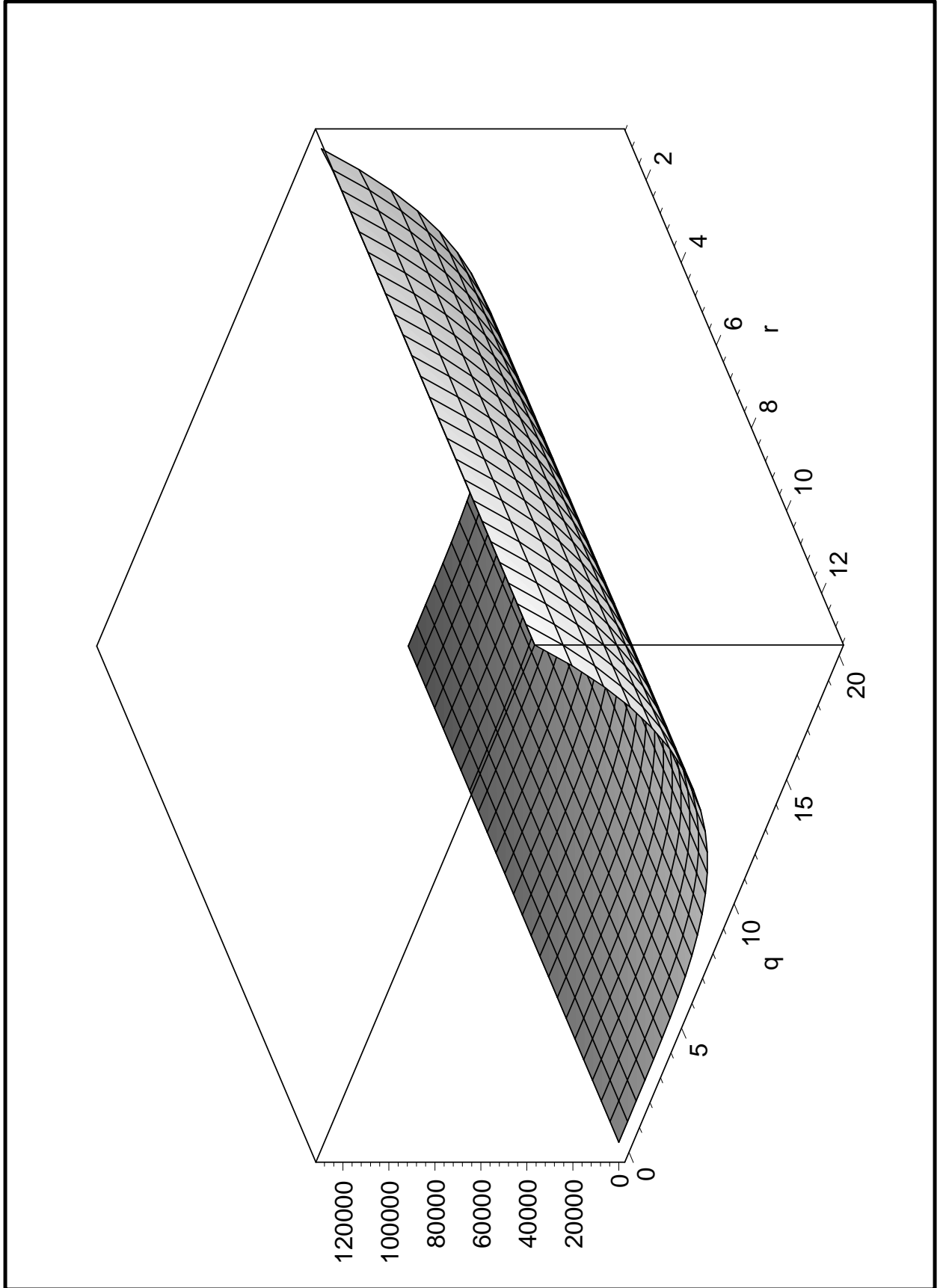


FIG. 4: This plot shows, in terms of R and q , the second derivative of the potential $V(R, q)$, for $\gamma = 0$ and for the intervals $0 \leq q \leq 20$ and $0 \leq R \leq 15$.

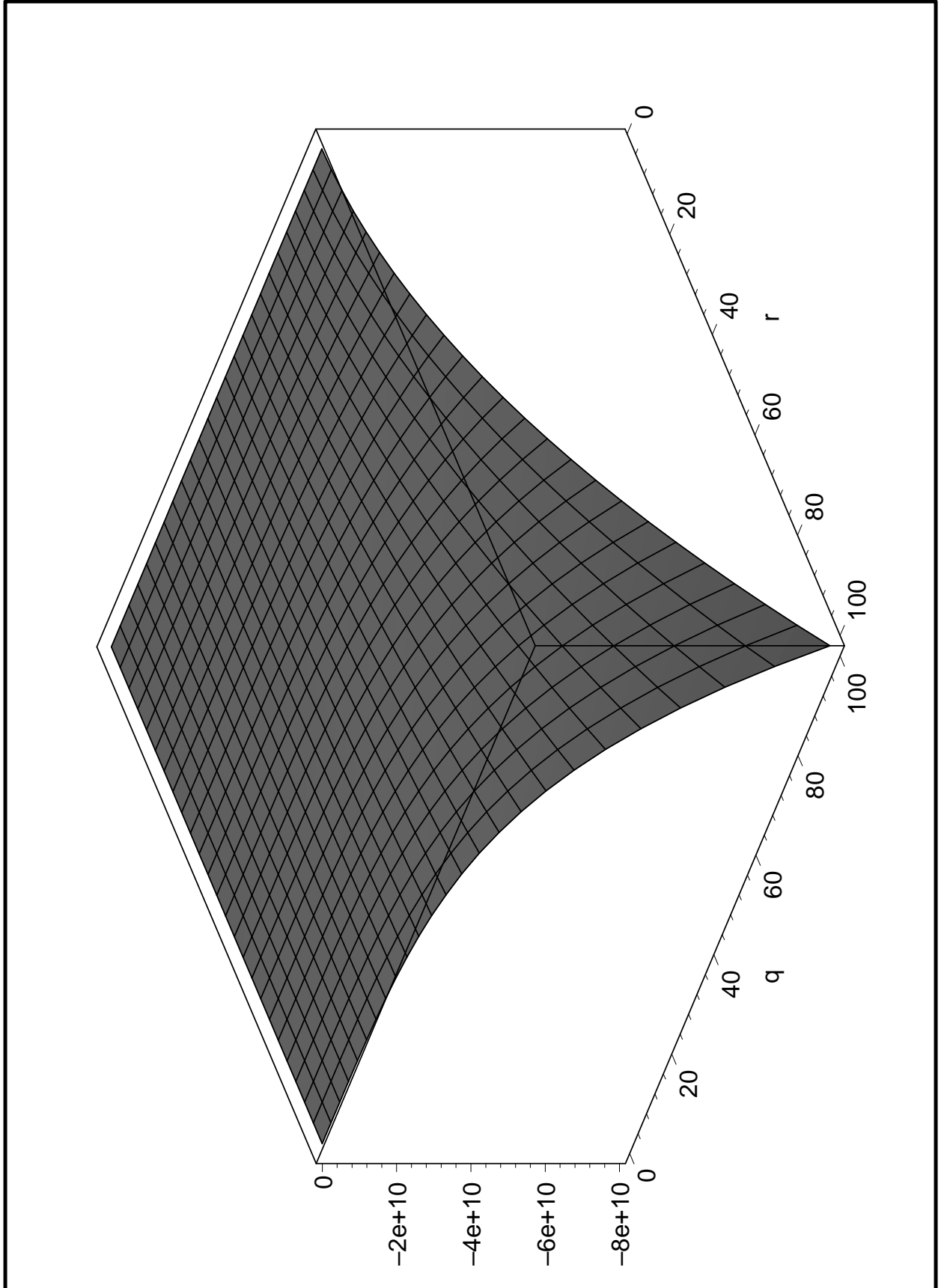


FIG. 5: This plot shows, in terms of R and q , the minimum of the potential $V(R, q)$, for $\gamma = 0$ for the intervals $0 \leq q \leq 100$ and $0 \leq R \leq 100$.

by

$$q_c = \frac{1}{\sqrt{6}} \sqrt{\frac{A^{2/3} + 92 R^6 + 52 + 4 R^{12} - 2 \sqrt[3]{A} R^6 + 10 \sqrt[3]{A}}{R^4 \sqrt[3]{A}}}, \quad (33)$$

where

$$A = 399 R^{12} + 1092 R^6 + 280 - 8 R^{18} + 3 \sqrt{3} \sqrt{-(16 R^{18} + 21 R^{12} + 48 R^6 + 16) (-12 + 5 R^6)^2}.$$

Note that there is only one possibility to have a real solution for $q_c = 1.918822876$, corresponding to the radius $R_c = \sqrt[6]{\frac{12}{5}} = 1.157093730$. Thus, for a stiff fluid shell there is an unique pair of the parameters R_c and q_c which is able to form a stable gravastar. The graphics of V' and V'' establish a minimum limit to the charge in order to have gravastar formation, which is reinforced by the absence of gravastar with zero mass and zero charge in previous paper [9]. Then, we can conclude that $q \geq q_c$ to form gravastar, where $q = q_c$ represents a stable gravastar, while $q > q_c$ represents bounded excursion gravastars.

These results allow us to conclude that the presence of charge in the shell created the necessary conditions to form a stable gravastar even with a zero Schwarzschild mass.

B. Dust Fluid Shell ($\gamma = 1$):

Now, we consider one shell constituted by dust fluid, meaning that we choose $\gamma = 1$. Again, vanishing of the first derivative of the potential gives us the critical values of the parameter L_i . Doing this we obtain

$$L_{ic}^2 = \frac{1}{\Delta_5} (R^4 q^2 + R^4 + 2 R^4 \Delta_4), \quad (34)$$

where $\Delta_4 = \sqrt{q^4 - q^2 + 1}$ and $\Delta_5 = -2 q^2 + 1 + q^4$. Putting this into the second derivative of the potential, we obtain the expression

$$\frac{d^2 V(R)}{dR^2} = \frac{1}{R^4 \Delta_6} (-8 + 24 q^2 - 8 q^8 + 24 q^6 - 32 q^4 - 4 q^6 \Delta_4 + 4 q^2 \Delta_4 + 4 q^4 \Delta_4 - 4 \Delta_4), \quad (35)$$

where $\Delta_6 = (q^2 + 1 + 2 \Delta_4)^2$.

Then we can see, from figures 6 and 7, that the second derivative of the potential is always negative. Thus, the potential of a charged dust shell cannot represent a stable (or even a bounded excursion) gravastar configuration, independently of the charge of the shell.

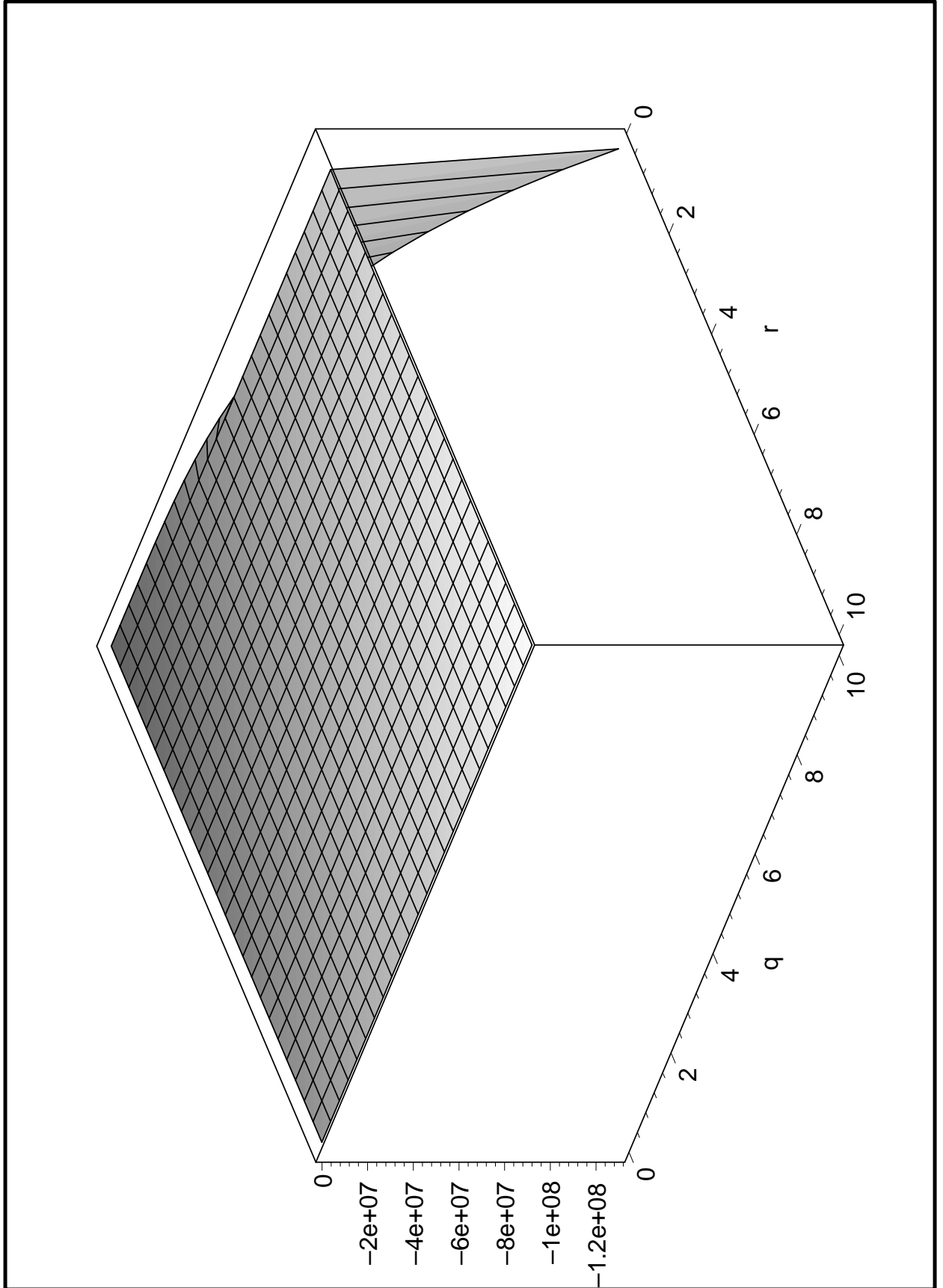


FIG. 6: This plot shows, in terms of R and q , the second derivative of the potential $V(R, q)$, for $\gamma = 1$ and for the intervals $0 \leq q \leq 10$ and $0 \leq R \leq 10$.

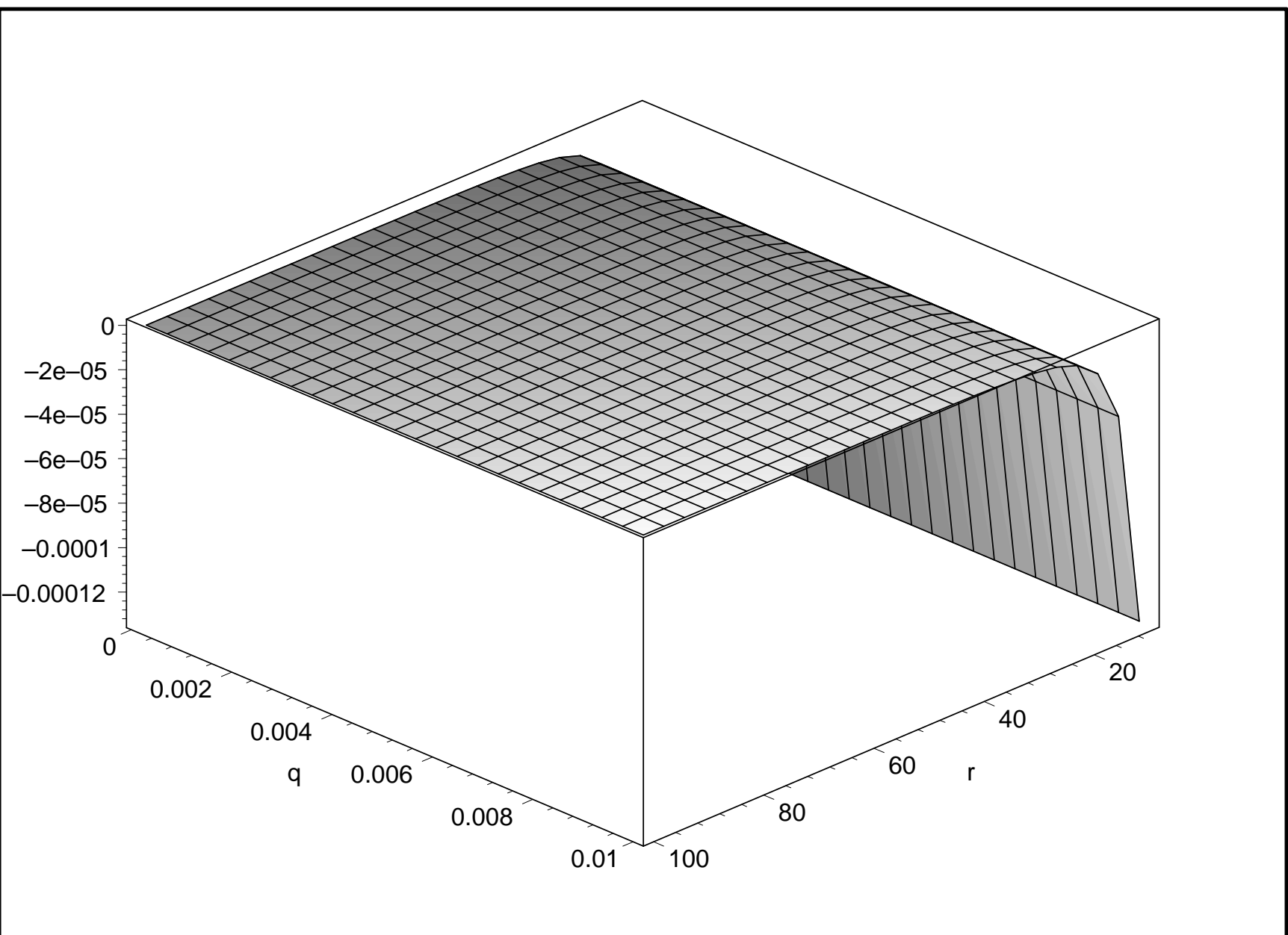


FIG. 7: This plot shows, in terms of R and q , the second derivative of the potential $V(R, q)$, for $\gamma = 1$ and for the intervals $0 \leq q \leq 0.01$ and $10^5 \leq R \leq 100$.

V. FINAL REMARKS

In this paper, we have generalized the problem of the stability of gravastars studied recently by us [9], introducing a charged shell with a Reissner-Nordström exterior spacetime. Thus, the model consists of a de Sitter interior spacetime, a dynamical infinitely thin shell of fluid with an equation of state $p = (1 - \gamma)\sigma$, and an external Schwarzschild spacetime.

In order to simplify the mathematical calculus and to isolate the effect of the charge in the gravastar formation, we investigated a particular solution which emerges from the gravastar studies with zero Schwarzschild mass, which implies in a non-gravitational object.

For the stiff fluid for the shell ($\gamma = 0$), we have shown that for $q \geq q_c$ it forms gravastar: when $q = q_c$ it forms a stable gravastar, while $q > q_c$ it forms bounded excursion gravastar.

For the dust shell ($\gamma = 1$) the potential of a charged dust shell cannot represent a stable (or even a bounded excursion) gravastar configuration, independently of the charge of the shell.

In summary, we have found that the presence of the charge contributes to the stability of the gravastar, if the charge is greater than a critical value. Otherwise, a massive non-gravitational object is formed for small charges.

Acknowledgments

The financial assistance from FAPERJ/UERJ (MFAdaS) is gratefully acknowledged. The author (RC) acknowledges the financial support from FAPERJ (no. E-26/171.754/2000, E-26/171.533/2002 and E-26/170.951/2006). MFAdaS and RC also acknowledge the financial support from Conselho Nacional de Desenvolvimento Científico e Tecnológico - CNPq - Brazil. The author (MFAdaS) also acknowledges the financial support from Financiadora de Estudos e Projetos - FINEP - Brazil (Ref. 2399/03).

-
- [1] D. Horvat and S. Ilijic, arXiv:0707.1636.
 - [2] P. Marecki, arXiv:gr-qc/0612178; F.S.N. Lobo, Phys. Rev. D **75**, 024023 (2007); arXiv:gr-qc/0612030; Class. Quantum Grav. **23**, 1525 (2006); F.S.N. Lobo, Aaron V. B. Arel-lano, *ibid.*, **24**, 1069 (2007); T. Faber, arXiv:gr-qc/0607029; C. Cattoen, arXiv:gr-qc/0606011;

- O.B. Zaslavskii, Phys. Lett. B**634**, 111 (2006); C. Cattoen, T. Faber, and M. Visser, Class. Quantum Grav. **22**, 4189 (2005).
- [3] E.J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D**15**, 1753 (2006); T. Padmanabhan, arXiv:07052533.
- [4] A.E. Broderick and R. Narayan, Class. Quantum Grav. **24**, 659 (2007) [arXiv:gr-qc/0701154].
- [5] P.O. Mazur and E. Mottola (2001) [arXiv:gr-qc/0109035].
- [6] M. Visser and D.L. Wiltshire, Class. Quantum Grav. **21**, 1135 (2004).
- [7] P. Rocha, A.Y. Miguelote, R. Chan, M.F. da Silva, N.O. Santos,, and A. Wang, J. Cosmol. Astropart. Phys. **6**, 25 (2008) [arXiv:gr-qc/08034200].
- [8] P. Rocha, R. Chan, M.F. da Silva and A. Wang, J. Cosmol. Astropart. Phys. **11**, 10 (2008) [arXiv:gr-qc/08094879].
- [9] R. Chan, M.F. da Silva, P. Rocha and A. Wang, J. Cosmol. Astropart. Phys. **3**, 10 (2009) [arXiv:gr-qc/08124924].
- [10] R. Chan, M.F. da Silva and P. Rocha, J. Cosmol. Astropart. Phys. **12**, 17 (2009) [arXiv:gr-qc/09102054].
- [11] R. Chan, M.F. da Silva and Jaime F. Villas da Rocha (2010) [arXiv:gr-qc/10042906].
- [12] B.M.N. Carter, Class. Quantum Grav. **22**, 4551 (2005).
- [13] D. Horvat, S. Ilijic and A. Marunovic, Class. Quant. Grav. **26**, 025003 (2009).
- [14] K. Lake, Phys. Rev. D **19**, 2847 (1979).
- [15] R. Adler, M. Bazin, M. Schiffer, *Introduction to General Relativity*, McGraw-Hill Inc., p. 489 (1975).
- [16] L. Marder, Proc. R. Soc. London, Ser. A, **224**, 524 (1958).
- [17] W. Israel, Physical Review **D 15**, 935 (1977).
- [18] A. Wang, M.F.A. da Silva and N.O. Santos, Class. Quantum Grav. **14**, 2417, (1997).